ELECTROMAGNETICS

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Theory
&
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1

BASICS OF ELECTROMAGNETIC THEORY & MAXWELL'S EQUATIONS

THEORY

1.1 Vector Algebra

There are 3-types of product

(i) Dot Product

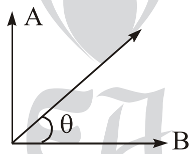
(ii) Cross Product

(iii) Triple Product

1.1.1 Vector Product

(i) Dot Product

The Dot Product of two Vectors $\vec{A}$ and $\vec{B}$ is given by,



$$\vec{A}\cdot\vec{B} = A\cdot B\cdot\cos\theta$$

Let,
$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$
$$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$$

Thus, Dot product is given by

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Dot product is a Scalar quantity.

(ii) Cross Product

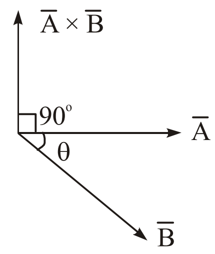
The Cross product of two Vectors $\vec{A}$ and $\vec{B}$ is given by

$$\vec{A}\times\vec{B} = |\vec{A}|\cdot|\vec{B}|\sin\theta\,\hat{a}_n$$

where, $\hat{a}_n$ = Normal unit vector. (Normal unit vector to AB plane)

$$\vec{A}\times\vec{B} = \hat{a}_x\left(A_yB_z - A_zB_y\right) - \hat{a}_y\left(A_xB_z - A_zB_x\right) + \hat{a}_z\left(A_xB_y - B_xA_y\right)$$

It is represented in the determinant form as given below



i.e.
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cross product is a Vector quantity.

(iii) Triple Product

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{C}.\vec{A}) - \vec{C}(\vec{A}.\vec{B})$$

1.1.2 Vector Operators

(1) Gradient Operator is ∇V of scalar V

(2) Divergence operator is $\nabla.\vec{V}$ of vector $\vec{V}$

(3) Curl $\nabla \times A$ of vector $\vec{A}$

(4) Laplacian $\nabla^2 V$ of scalar V

(1) Gradient ($\vec{\nabla}$ operator) : Operator is given by ∇V

Here del operator
$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

Gradient is applicable for Scalar fields only.

It given the Rate of change of Scalar field along the different co-ordinate axes.

Example : Gradient of potential field V is given by

$$\vec{\nabla} V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

where, V is a Scalar field.

Note : *Gradient of a potential field gives the electric field.*

i.e.,
$$\vec{E} = -\vec{\nabla} \cdot V$$
, where, E is the electric field intensity.

Note : *Gradient of a scalar field is a Vector quantity.*

(2) Divergence : It is applicable for a Vector field. Divergence of a Vector field gives the flux coming out of a closed surface, when volume of the surface shrinks to zero.

Let,
$$\vec{D} = D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z$$
= Electric flux density

$$\vec{\nabla}\cdot\vec{D} = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\cdot\left(D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z\right)$$

$$\therefore \quad \vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$$

The above equation represent the divergence of a Vector quantity $(\vec{D})$.

Note : *Divergence of a Vector field is a Scalar field.*

Example : $\vec{\nabla}\cdot\vec{D} = \rho_v$ = Charge density

(3) Curl of a Vector field : The curl of a Vector field.

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z \text{ is given by}$$

$$\vec{\nabla}\times\vec{A} = \left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right)\hat{a}_x - \left(\frac{\partial}{\partial x}A_z - \frac{\partial}{\partial z}A_x\right)\hat{a}_y + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{a}_z$$

For example, The curl of Magnetic field intensity $(\vec{H})$ represented as

$$\vec{H} = H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z$$

Can be given by determinant form as shown below

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}H_z - \frac{\partial}{\partial z}H_y\right)\hat{a}_x - \left(\frac{\partial}{\partial x}H_z - \frac{\partial}{\partial z}H_x\right)\hat{a}_y + \left(\frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x\right)\hat{a}_z$$

Note : *Curl of a Vector field is a Vector quantity.*

Example : $\vec{\nabla}\times\vec{H} = J$ = Current density.

(4) Laplacian (∇^2) : Laplacian of scalar V is divergence of gradient of scalar V

$$\nabla^2 V = \nabla.\nabla V = \text{Divergence (Gradient V)}$$

For Cartesian Coordinate

Laplacian $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Note : *A vector* $\vec{A}$ *is said to be solenoidal if its divergence is zero*

$$\nabla.\vec{A} = 0$$

Example : Magnetic field is solenoidal

$$\nabla.\vec{B} = 0$$

A vector $\vec{A}$ is said to be irrotational if its curl is zero.

$$\nabla \times \vec{A} = 0$$

Example : In static environment Electric field is irrotational or conservative

$$\nabla \times E = 0$$

- *A scalar field is said to be harmonic in given region if its laplacian is zero.*

$$\nabla^2 V = 0$$

- *Divergence of curl is always zero*

$$\nabla.(\nabla \times A) = 0$$

- *Curl of gradient is always zero*

$$\nabla \times (\nabla.A) = 0$$

1.1.3 Divergence Theorem

According the divergence theorem, "The surface integral of a vector field over a closed surface S is equal to the Volume integral of divergence of the Vector field".

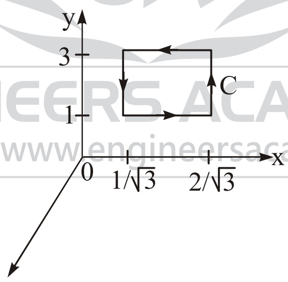
$$\oint_s \vec{D} \cdot \overrightarrow{ds} = \int_v \vec{\nabla} \cdot \vec{D}\, dv$$

1.1.4 Stokes Theorem

According to this theorem, "The Line integral of a Vector field over a closed path is equal to the Surface integral of curl of the Vector field".

$$\oint_\ell \vec{H} \cdot \overrightarrow{d\ell} = \int_s (\vec{\nabla} \times \vec{H}) \cdot \overrightarrow{ds}$$

Example 1 : Given vector field $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$. Find $\oint_c A.dl$ circulation by stoke's theorem over path given below.



Solution :

Stoke's theorem $\qquad \oint_c A.dl = \iint (\nabla \times A) ds$

Curl $\qquad \nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & x^2 & 0 \end{vmatrix}$

$$= \left[\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(x^2)\right]a_x + \left[\frac{\partial}{\partial z}(xy) - 0\right]a_y + \left[\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy)\right]a_z$$

$$\nabla \times A = x.\hat{a}_z$$

area element $\quad ds = dx\ .\ dy\ .\ \hat{a}_z$

Using stokes' theorem

$$\oint A.dl = \iint (\nabla \times A).ds$$

$$= \int_1^3 \int_{1/\sqrt{3}}^{2/\sqrt{3}} x.dx\,dy = 1$$

Example 2 : Given the vector field $A = y^2 a_x + (2xy + x^2 + z^2)\, a_y + (4x + 2yz)\, a_z$. Find divergence of vector field.

Solution :

Divergence is given by $\quad \nabla.\vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$

$$= \frac{\partial}{\partial x}[y^2] + \frac{\partial}{\partial y}[2xy + x^2 + z^2] + \frac{\partial}{\partial z}[4x + 2yz]$$

$$= 0 + 2x + 2y$$

$$= 2(x + y)$$

Example 3 : A scalar field $g = (1 + 2k)x^2y + xyz$ will be harmonic at all point for which value of k.

Solution :

Condition for harmonic field $\nabla^2 g = 0$

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} = 0$$

$$= \frac{\partial}{\partial x}[2x(1+2k)y + yz] + \frac{\partial}{\partial y}[(1+2k)x^2 + xz] + \frac{\partial}{\partial z}[xy]$$

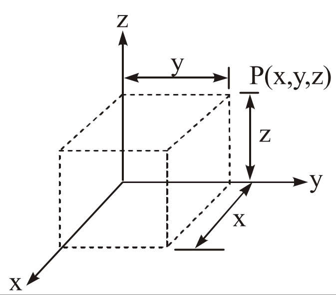
$$= 2(1+2k) + 0 + 0 = 0$$

Therefore $\quad k = -\frac{1}{2}$

1.2 Co-ordinate Systems

1.2.1 Cartesian Co-ordinate System

The co-ordinates of a point P in the cartesian co-ordinate system is x, y and z along the x, y and z axis. It can be represented as P (x, y, z) as shown below



- Differential length in cartesian coordinate is

$$\overrightarrow{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

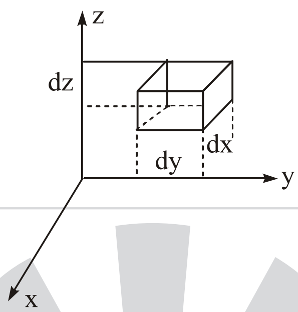
- Differential area in cartesian co-ordinates

$$\overrightarrow{ds_1} = dy\,dz \cdot \hat{a}_x$$

$$\overrightarrow{ds_2} = dx\,dz \cdot \hat{a}_y$$

$$\overrightarrow{ds_3} = dx\,dy \cdot \hat{a}_z$$

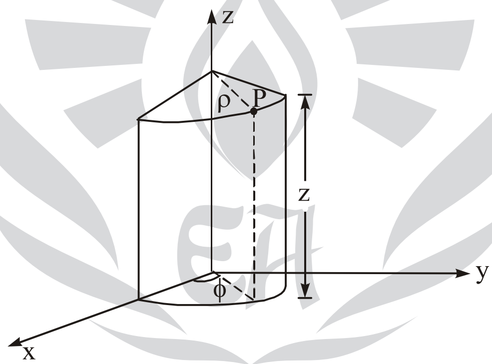
- Differential volume in cartesian co-ordinates is



$$dv = dxdydz$$

1.2.2 Cylindrical Co-ordinate System

The cylindrical co-ordinates of a point is represented in terms of ρ, ϕ and z along the cylinder as given below



Here, ρ = Radius of cylinder.

ϕ = Angle between x-axis and perpendicular on x-axis of the point.

- Differential volume in cylindrical co-ordinates is given by

$$dv = \rho d\rho . d\phi . dz$$

- Differential Length in cylindrical co-ordinates is given by

$$d\ell = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z$$

- Differential Area in cylindrical co-ordinates is given by

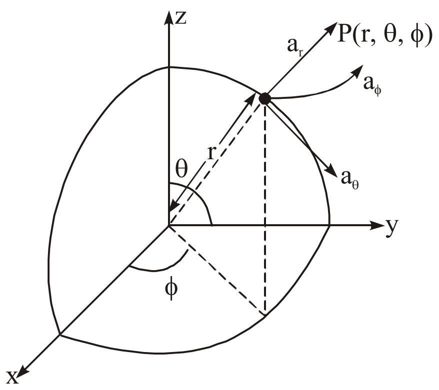
$$\overrightarrow{ds_\rho} = \rho d\phi dz \cdot \hat{a}_\rho$$

$$\overrightarrow{ds_\phi} = d\rho \cdot dz \cdot \hat{a}_\phi$$

$$\overrightarrow{ds_z} = (d\rho)(\rho d\phi) \cdot \hat{a}_z$$

1.2.3 Spherical Co-ordinate System

The spherical co-ordinates of a point is represented interms of r, θ & ϕ along the spherical surface as shown below :



Differential length

$$dl = dr\,\hat{a}_r + rd\theta\,\hat{a}_\theta + r\sin\theta.d\phi\,\hat{a}_\phi$$

Differential area

$$ds = \begin{cases} r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r \\ r\sin\theta\,dr\,d\phi\,\hat{a}_\theta \\ r\,dr\,d\theta\,\hat{a}_\phi \end{cases}$$

Differential volume $\quad dv = (dr)\ (rd\theta)\ (r\ \sin\ \theta\ d\phi)$

1.2.4 General Co-ordinate System : (U, V, W)

	U	**V**	**W**	**h_1**	**h_2**	**h_3**
Rectangular	x	y	z	1	1	1
Cylindrical	ρ	ϕ	z	1	ρ	1
Spherical	r	θ	ϕ	1	r	r sin θ

Mathematical Expressions of Operators

(i) Gradient $\quad \nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial V}{\partial v} + \frac{1}{h_3}\frac{\partial V}{\partial w}$

(ii) Divergence of Vector $\quad \vec{A} = A_u\hat{a}_u + A_v\hat{a}_v + A_w\hat{a}_w$

$$\nabla.\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}[h_2h_3A_u] + \frac{\partial}{\partial v}(h_3h_1A_v) + \frac{\partial}{\partial w}(h_1h_2A_w)\right]$$

(iii) Laplacian $\quad \nabla^2 V = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3h_1}{h_2}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial w}\left(\frac{h_1h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$

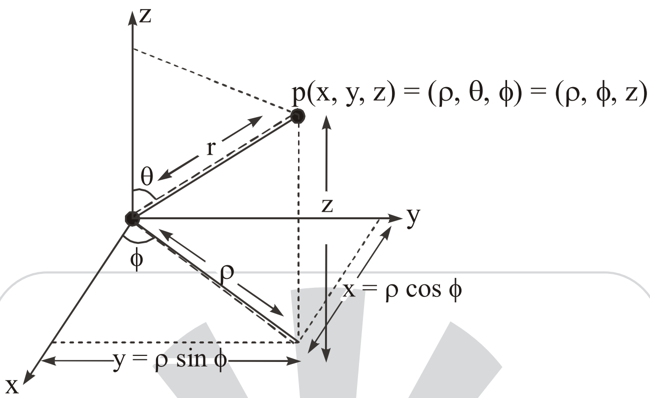
(iv) Curl $\quad \nabla\times\vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{a}_u & h_2\hat{a}_v & h_3\hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1A_u & h_2A_v & h_3A_w \end{vmatrix}$

(v) Area $$ds = \begin{cases} h_2h_3 \ \partial v \partial w \ \hat{a}_u \\ h_1h_3 \ \partial u \partial w \ \hat{a}_v \\ h_1h_2 \ \partial v \partial w \ \hat{a}_w \end{cases}$$

(vi) Volume $$dv = h_1 h_2 h_3 \partial u \, \partial v \, \partial w$$

(vii) Length $$dl = h_1 du \, \hat{a}_u + h_2 dv \, \hat{a}_v + h_3 dw \, \hat{a}_w$$

1.2.5 Co-ordinate Transformation



Relation between Cylindrical and cartesian co-ordinates

Cylindrical	***Cartesian***
$\rho = \sqrt{x^2 + y^2}$	$x = \rho \cos \phi$
$\phi = \tan^{-1}\left[\dfrac{y}{x}\right]$	$y = \rho \sin \phi$
$z = z$	

Relation between spherical and other co-ordinates

Spherical	***Other Co-ordinates***
$r = \sqrt{x^2 + y^2 + z^2}$	$\rho = r \sin \theta$
$\theta = \tan^{-1}\left(\dfrac{\rho}{z}\right) = \tan^{-1}\left(\dfrac{\sqrt{x^2+y^2}}{z}\right]$	$x = r \sin \theta \cos \phi$
$\phi = \tan^{-1}\left(\dfrac{y}{x}\right)$	$y = r \sin \theta \sin \phi$

Example 4 : Determine divergence of vector fields

(a) $\vec{A} = \rho \sin \phi \ \hat{a}_\rho + \rho^2 z \, \hat{a}_\phi + z \cos\phi \, \hat{a}_z$

(b) $\vec{B} = \dfrac{1}{r^2} \cos \theta \, \hat{a}_r + r \sin^2 \theta \cos\phi \, \hat{a}_\theta + \cos\theta \, \hat{a}_\phi$

Solution :

(a) $$\nabla.\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(A_\phi) + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{\rho}\frac{1}{\partial\rho}(\rho^2 \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho^2 z) + \frac{\partial}{\partial z}[z\cos\phi]$$

$$= 2\sin\phi + \cos\phi$$

(b)
$$\nabla.\vec{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(B_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(B_\phi)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\cos\theta)$$

$$= 0 + 2\cos\theta\cos\phi + 0$$

$$= 2\cos\theta\cos\phi$$

Example 5 : For above vector field $\vec{A}$ find curl $\nabla\times\vec{A}$

Solution :

$$\nabla\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \rho\sin\phi & \rho^2 z & z\cos\phi \end{vmatrix}$$

$$= \left[\frac{z}{\rho}\sin\phi - \rho^2\right]\hat{a}_\rho + 0 + \frac{1}{\rho}[3\rho^2 z - \rho\cos\phi]\hat{a}_z$$

$$= -\frac{1}{\rho}(z\sin\phi + \rho^3).\hat{a}_\rho + (3\rho z - \cos\phi)a_z$$

1.3 Electrostatics

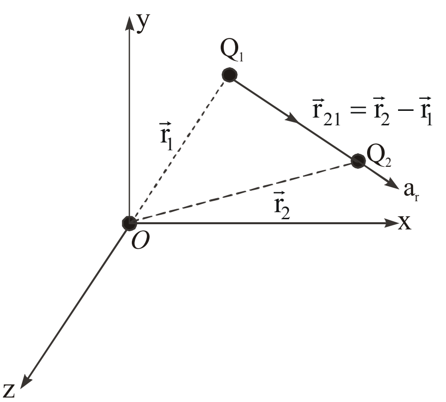
Stationary charge produces electric field $\vec{E}$.

A charge may be point charge, line charge, surface charge or volume charge distributed.

There are two laws in electrostatics **coulomb's law and gauss law.**

1.3.1 Coulomb's Law

Statement : The force between two point charge Q_1 and Q_2 is inversely proportional to square of distance between two charges and directed along the vector connecting two charges.



Force,
$$F = \frac{kQ_1Q_2}{|\vec{r}_{21}|^2}.\hat{a}_r$$

$$F_{21} = \frac{Q_1Q_2.(\vec{r}_2 - \vec{r}_1)}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|^3}$$

Electric field $\vec{E}$ intensity is defined as force per unit charge

$$\vec{E} = \frac{\vec{F}}{Q}$$

- Electric field due to point charge

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}.\hat{a}_r$$

- Electric field due to line charge

$$\vec{E} = \frac{\int \rho_L dl}{4\pi\epsilon_0 r^2}\hat{a}_r$$

- Electric field due to surface charge

$$\vec{E} = \frac{\iint \rho_s ds}{4\pi\epsilon_0 r^2}.\hat{a}_r$$

- Electric field due to volume charge

$$\vec{E} = \frac{\iiint \rho_v dv}{4\pi\epsilon_0 r^2}.\hat{a}_r$$

Electrostatic potential is defined as work done per unit charge and it is scalar potential due to point charge.

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Gradient of potential is electric field.

$$\vec{E} = -\nabla V$$

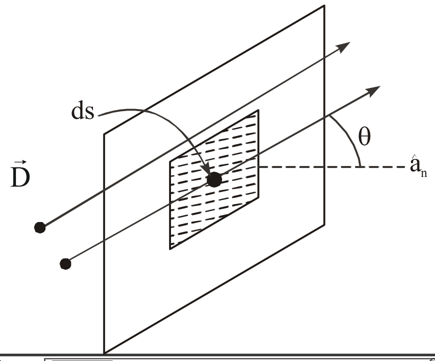
For close loop 'C' work done is zero

$$V = -\oint_C E.dl = 0$$

by stokes theorem for static field.

$$\nabla \times \vec{E} = 0$$

Electric flux passing through any surface areas



Electric flux $\Psi = \iint D.ds$

where, D = Electric field density c/m^2.

1.3.2 Gauss's Law

Statement : The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

i.e., $$\Psi = \oint_S D.ds = Q_{enclosed}$$

Integral form $$\oint_S D.ds = \int_v \rho_v dv \qquad (\rho \neq \rho_v)$$

ρ_v = Volume charge density

Differential form $$\nabla.D = \rho_v$$

Example 6 : Charge density inside a hollow spherical shell of radius r = 4 cm centered at origin defined as

$$\rho_v = \begin{cases} 0 & \text{for} \quad r \leq 2 \\ \dfrac{4}{r^2} c/m^3 & \text{for} \quad 2 < r \leq 4 \end{cases}$$

Find Electric field intensity at r = 3

Solution :

From Gauss law $$\oint E.ds = \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho_v dv$$

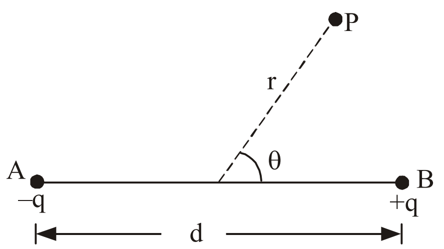
$$= \frac{1}{\epsilon_0}\int (0)dv + \frac{1}{\epsilon_0}\int \frac{4}{r^2}dv \qquad [0 < r \leq 3]$$

$$E(4\pi R^2) = \frac{1}{\epsilon_0}\int_{r=2}^{3}\int_0^{\pi}\int_0^{2\pi} \frac{4}{r^2}\,[r^2 \sin\theta \; dr \; d\theta \; d\phi]$$

$$E\,(4\pi \times 3^2) = \frac{4\pi \times 4}{\epsilon_0}(3-2)$$

$$E = \frac{4}{9\epsilon_0} a_r$$

1.3.3 Electric Dipole



- Electric dipole consist of two point charge, separated by small distance d having opposite polarity.
- Dipole moment $p = qd$
- Electric potential due to dipole is given by

$$V = \frac{P\cos\theta}{4\pi \in_0 r^2}$$

Note : *Potential is maximum along dipole and it is inversely proportional to square of distance.*
Electric field due to dipole is given by

$$\vec{E} = \frac{P}{4\pi \in_0 r^3}[2\cos\theta\, a_r + \sin\theta\, a_\theta]$$

Note : *For monopole* $\vec{E} \ \alpha \ \frac{1}{r^2}$

For Dipole $\vec{E} \ \alpha \ \frac{1}{r^3}$

1.3.4 Electrostatic Energy

Energy stored in the system with electric field E and electric flux density $\vec{D}$ is given by

$$W_e = \frac{1}{2}\int_v \vec{D}.\vec{E}.dv$$

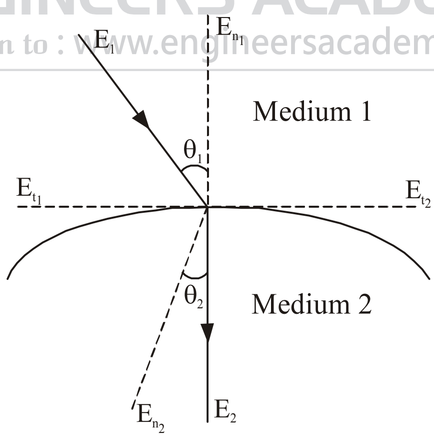
$$= \frac{1}{2}\int_v \in_0 E^2 dv$$

1.3.5 Electric Boundary Conditions

Boundary conditions are defined when region consist of two different media.
Electric field composed of two orthogonal component, tangential component E_t and normal component E_n.

$$E = E_t + E_n$$

Consider the two different dielectric medium (1) and (2) with permittivities $\in_1$ and $\in_2$ respectively as shown below



According to boundary condition, tangential component of electric field is continuous at boundary, i.e., $E_{t_1} = E_{t_2}$ or $\frac{D_{t_1}}{\epsilon_1} = \frac{D_{t_2}}{\epsilon_2}$

If the surface charge density at boundary is e_s then boundary condition becomes.

$$D_{n_1} - D_{n_2} = e_s$$

1.3.6 Poission's and Laplace Equation

Electric potential V and volume charge density E_v in certain region is related by poissions equation

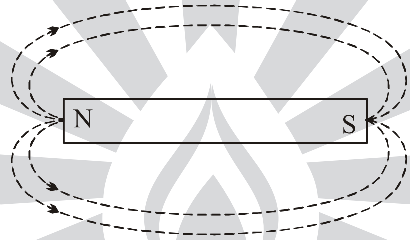
$$\nabla^2 V = \frac{\rho_V}{\epsilon}$$

For charge free region $\nabla^2 V = 0$

Uniqueness theorem states that if solution of Laplace or poission equation satisfies the boundary condition, then solution is 'Unique'.

1.4 Magnetostatic Fields

Magnetic field is produced by moving charges or constant current flow.



Magnetic flux is concentration of magnetic flux line outward from north pole towards south pole of magnet.

Magnetic flux density is defined as magnetic flux per unit area and it is vector quantity. Its unit is Tesla (T) or weber per squared meter (1 wb/m^2)

$$\vec{B} = \frac{d\Psi}{ds} a_n$$

Flux $\Psi = \int_s B.ds$

Relation between magnetic flux density $\vec{B}$ and magnetic field intensity $\vec{H}$ is given by

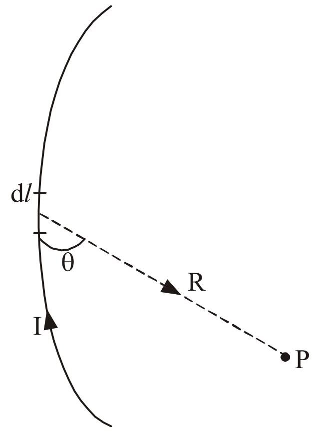
$$B = \mu H = \mu_0 \mu_r H$$

where, μ = Permeability of medium

$\mu_0 = 4\pi \times 10^{-7} H/m$

1.4.1 Bio-Savart's Law

Statement : The magnetic field intensity dH produces at point P due to current element Id*l* is given by



$$dH = \frac{Idl \sin\theta}{4\pi R^2}$$

or
$$dH = \frac{Idl \times a_R}{4\pi R^2} = \frac{Idl \times \vec{R}}{4\pi R^3}$$

Direction of magnetic field is given by right-hand rule where fingers shows magnetic field line and direction of thumb show current I.

1.4.2 Ampere's Law

Statements : Line integral of magnetic field intensity around any closed path is equal to current enclosed by the path

$$\oint_l H.dl = I_{enclosed}$$

By stoke's theorem

$$\oint_l H.dl = \iint_s (\nabla \times H).ds = \iint J.ds$$

or
$$\nabla \times H = J$$

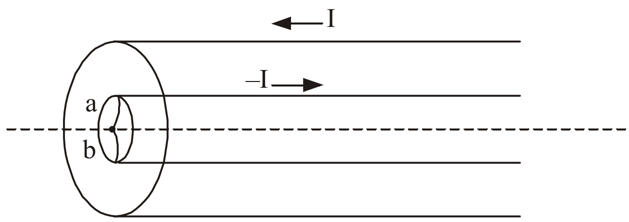
Curl of magnetic field intensity $\vec{H}$ is equal to current density J.

Example 7 : Consider hollow concentric cylinder carrying I and –I current in opposite direction. Find magnetic field intensity inside and outside cylinder.

Solution :

Case-I : If $r < a$

using ampere's Law



$$\oint H.dl = I_{enc.}$$

Inside inner cylinder current enclosed is zero

$$\oint H.dl = 0$$

$$H = 0 \text{ inside inner cylinder.}$$

Case-II : If $a < r < b$

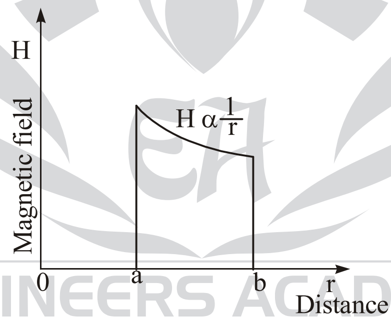
$$\oint H.dl = I$$

$$H = \frac{I}{2\pi r^2} a_\phi$$

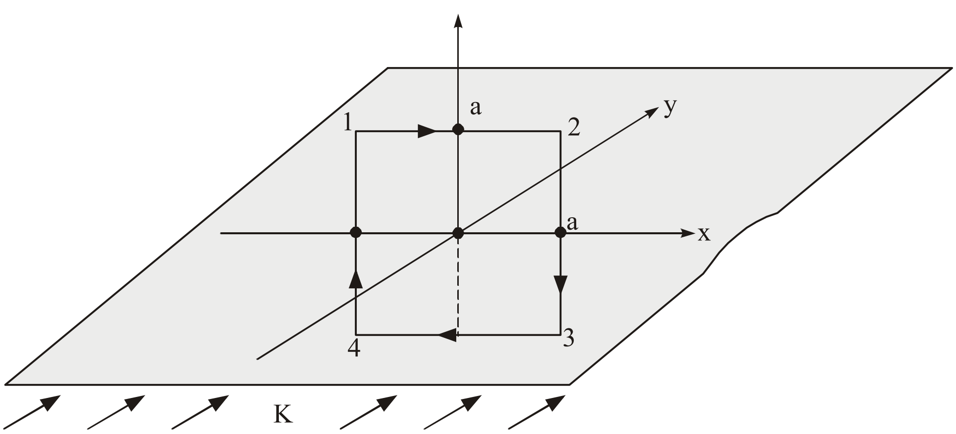
Case-III : If $r > b$

$$\oint H.dl = I - I = 0$$

$$H = 0 \text{ outside outer cylinder}$$



Example 8 : An infinite current sheet lies in the $z = 0$ plane with $K = ka_y$, as shown in figure. Find H.



Solution :

The Biot-Savart law and considerations of symmetry shown that H has only an x component, and is not a function of x or y.

Apply Ampere's Law to the square contour 2341, and using the fact that H must be antisymmetric in z,

$$\oint H.dl = (H)(2a)+0+(H)(2a)+0$$

$$= (K)(2a) \quad \text{or} \quad H = \frac{K}{2}$$

Thus for all z > 0, $$H = \left(\frac{K}{2}\right)a_x.$$

More generally, for an arbitrary orientation of the current sheet,

$$H = \frac{1}{2}K \times a_n$$

a_n is the unit vector perpendicular to the plane of the sheet.

Observe that H is independent of the distance from the sheet. Further, the directions of H above and below the sheet can be found by applying the right-hand rule to a few of the current elements in the sheet.

1.4.3 Magnetic Potential

There are two type of magnetic potentials

(1) Magnetic scalar potential (V_m)

$$H = -\nabla V_m$$

or $$(V_m) = \int_y^x H.dl$$

For source free region (J = 0), then magnetic scalar potential satisfies the Laplace's equation

i.e., $$\nabla^2 V_m = 0$$

It is only defined for current free region.

(2) Magnetic Vector Potential $(\vec{A})$:

Magnetic field density $\vec{B}$ can be expressed as curl of magnetic vector potential $\vec{A}$

$$\vec{B} = \nabla \times \vec{A}$$

Magnetic vector potential satisfies the poission's equation

i.e., $$\nabla^2 A = -\mu_0 J$$

Example 9 : Find current density that would produce magnetic vector potential $A = 2a_\phi$ in cylindrical co-ordinate.

Solution :

Magnetic flux density is given by

$$B = \nabla \times A$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\phi)a_z$$